Can one reconstruct the

seesaw parameters?

(the masses, mixing matrix and Yukawa couplings of the $\nu_R$)

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based mostly on JHEP 0109 (2001) 013, with A Ibarra
Why is this an interesting question?

- the \((3 \, \nu_R, \text{no triplet})\) seesaw today...
  - we observe small neutrino masses
  - "attractive" mechanism consistent with small \(m_\nu\)

- the seesaw predicts
  - \(m_\nu\) \text{ majorana (0}\nu\text{2}\beta)\)
  - if no SUSY (LHC?), then flavour, \(\text{CP} \propto m_\nu\):
    \(\mu \rightarrow e\gamma\) small, EDMs small, \(\nu\) mag. mos small
  - with SUSY:

\[\text{can be observable}\]

these are consistency checks

- to "test" a model, one needs
  1. determine model parameters from data
  2. make a prediction for an additional measurement
  3. confirm prediction

\(\Rightarrow\) \text{can seesaw parameters be determined from data?}\
NO

(realistic answer)

...but in the best of all (SUSY) dream worlds ??? ...yes...

⇒ HOW? and WHY DOES IT WORK?
Outline

- parametrisations of the (type I) seesaw :
  - RH parameters (usual top-down)
  - LH parameters (? “bottom-up” ?)
  - (à la Casas-Ibarra)
  - fallacies thereof... the puppet show?

  ⇒ what is special about the seesaw?

- “reconstruction” in SUSY
  - in principle (LH → RH)
  - in practise...

- “reconstruction” without SUSY? (Broncano et al)

- (leptogenesis as a “test”?)

- summary
input RH parameters

the usual seesaw parametrisation (3 $\nu_R$, no triplet scalars):

1. 3 $\nu_R$ with masses $M_i$
2. a Yukawa $[Y_\nu]_{RL}$ with eigenvalues $\{y_1, y_2, y_3\}$
3. a unitary matrix $[V_R] : D_{Y_\nu} \leftrightarrow D_M$
4. $Y_e$ eigenvalues $\{y_e, y_\mu, y_\tau\}$
5. a unitary matrix $V_L : D_{Y_e} \leftrightarrow D_{Y_\nu}$, (or, $U : D_Y \leftrightarrow D_{m_\nu}$)

\[ \nu_R \text{ bases : } D_{Y_\nu}, \quad D_M \quad \nu_L \text{ bases : } D_{Y_\nu}, \quad D_{Y_e} \]
parameters: 9 real eigenvals, 6 angles, 6 phases

\[ \Rightarrow [m_\nu] = V_L^T D_Y V_R^* D_M^{-1} V_R^\dagger V_Y V_L^\dagger (Y_e \text{ basis}) \]
\[ = U^* D_{m} U^\dagger \]

so in summary:

RH inputs: $D_M, [V_R], D_{Y_\nu}$, also LH inputs $[V_L], D_{Y_e}$
parameters: $12 + 9 = 21$.
can calculate: $U, D_m$
Input LH parameters

The seesaw treats $\nu_L$ and $\nu_R$ symmetrically... so:

1. 3 $\nu_L$ with masses $m_i$
2. a Yukawa $[Y_L]_{RL}$ with eigenvalues $\{y_1, y_2, y_3\}$
3. a unitary matrix $[W_L]$ : $D_{Y_L} \leftrightarrow D_m$
4. $Y_e$ eigenvalues $\{y_e, y_\mu, y_\tau\}$
5. a unitary matrix $V_L : D_{Y_e} \rightarrow D_{Y_L}$, or, $U : D_{Y_e} \rightarrow D_{m_N}$

\[ M^{-1} = D_{Y_L}^{-1}W_L^* D_m W_L^\dagger D_{Y_L}^{-1} = V_R^* D_M^{-1} V_R^\dagger \]

In summary:

LH inputs: $D_m, [W_L], D_{Y_L}, U, D_{Y_e}$
parameters: $12 + 9 = 21$

can calculate $D_M, V_R$

LH particle masses are $\lesssim m_W$...so...

1) do weak-scale observables depend on $W_L (= V_L U)$ and $D_Y$ ?
2) is there any (realistic) hope of extracting $V_L$ and $D_Y$ from data?
conclusions one should not draw (d’après moi)

1. “leptogenesis is independent of LH parameters”
   proof:
   \[ \epsilon = \sum_{J} \frac{\Im \{ [V_R^\dagger D_Y^2 V_R]_{1J} \} g \left( \frac{M_j^2}{M_1^2} \right)}{[V_R^\dagger D_Y^2 V_R]_{11}} \]
   no LH parameters appear. QED.

2. “light ν observables are independent of RH parameters”
   proof:
   \[ P(\nu_\beta \rightarrow \nu_\alpha) = f(U_{\beta k}, U_{\alpha j}, ..., m_j^2 - m_k^2) \]
   no RH parameters appear. QED.

1. \( D_m \) and \( U \) can be calculated from \( \nu_R \) parameters, so obviously depend on them. Whereas leptogenesis does not care about \( Y_\nu \), so does not care about \( U \). Explicitly: \( U \) does not appear in the formula for \( \epsilon \) in any LH parametrisation that includes \( W_L : D_{Y_\nu} \rightarrow D_m \).

2. but — in the “top-down” parametrisation using \( (D_M, V_R, D_{Y_\nu}, U, D_{Y_e}) \), \( V_R \) does not appear in \( P(\nu_\beta \rightarrow \nu_\alpha) \).

1. specious argument! At \( \Lambda \sim M \), \( D_{Y_e} \) and \( D_{Y_\nu} \) are coupling constants in the LH sector. \( D_m \) is not. So a “correct” top-down parametrisation uses \( V_L \) not \( U \).

2. ah-ha! but at \( \Lambda \sim m_W \) we know \( U \) and might get \( V_L \) in SUSY, so \( W_L = V_L U \) is dependent in LH parametrisation. So \( \epsilon(U) \) ...

what is correct choice of “independent” parameters?

\[ \begin{align*}
D_m & \leftrightarrow U \\
 & \rightarrow D_{Y_e} \leftrightarrow V_L \rightarrow D_{Y_\nu}
\end{align*} \]
the SUSY See-Saw

- suppose soft scalar masses universal at $M_{GUT}$: $\sim m_o^2 I$
- Renormalisation Group running will induce flavour violation at the weak scale in slepton masses:

\[
\begin{align*}
\begin{array}{c}
\nu_R \\
\tilde{\nu}_i \cdots \bar{Y}_\nu \cdots Y_\nu \cdots \tilde{\nu}_j \\
\tilde{h}
\end{array}
\end{align*}
\]

\[
[m^2_{\tilde{u}}]_{ij} \simeq (\text{diag part}) - \frac{3m_0^2 + A_0^2}{8\pi^2} (Y_\nu^\dagger)_{ik} (Y_\nu)_{kj} \log \frac{M_{GUT}}{M_k}
\]

$Y_\nu^\dagger Y_\nu$ is the only source of lepton flavour violation in $[m^2_{\tilde{u}}]_{ij}$

$\Rightarrow$ extract $Y_\nu^\dagger Y_\nu \propto [V_L]^\dagger D_Y^2 [V_L]$ from $[m^2_{\tilde{u}}]$ (in principle)

$\Rightarrow \{D_{Y_\nu}, V_L\}$ from sleptons, $\{D_{Y_e}, U, D_m\}$ from leptons
reconstruction in practice

from the leptons

- $D_{\ell e} : m_\tau, m_\mu, m_e$
  (could determine $\tan \beta$ in SUSY?)

- $U : \theta_{23}, \theta_{12}, \theta_{13}, \delta, \alpha, \beta$
  with a pure, high intensity $\nu$ beam: $\theta_{13}, \delta$?

- $D_{m} : m_3 > m_2 > m_1$
  know two $\Delta m^2$s, need pattern and absolute mass scale
  $0 \nu 2\beta$, cosmology?

from the sleptons

- $V_L : |[V_L]_{23}|, |[V_L]_{23}|, |[V_L]_{12}|$
  
  if superpartners are “light” (observable at LHC, in $\ell_j \rightarrow \ell_i \gamma$)
  then from LFV, might extract $[\tilde{m}_e^2]_{\tau \mu}, [\tilde{m}_e^2]_{\mu e}, [\tilde{m}_\tau^2]_{\tau e}$

  if SUSY at $\Lambda > M_3$, then $[\tilde{m}_e^2]_{\alpha \beta} \sim [V_L]_{\alpha 3}[V_L]_{\beta 3} y_3^2$
  \[ \Rightarrow \text{extract } ? |[V_L]_{23}|, |[V_L]_{13}|, y_3 \]
  ACK—other contributions to non-universal $[\tilde{m}_e^2]$ ??

- $D_{y_i} : y_3, y_2, y_1$
  ...only $y_3$ makes significant contribution to RGEs

- 3 phases of $V_L : \varphi_1, \varphi_2, \varphi_3$
  CP in charged sleptons? $\varphi$EDMs $\varphi$, $\varphi \nu - \bar{\nu}$ oscillations $\varphi$
  but there are other sources of CP that can contribute...
Summary

the (type I) seesaw treats $\nu_L$ and $\nu_R$ symmetrically, so can be described by masses, mixing angles and couplings of $\nu_R$, or $\nu_L$.

LH particles are kinematically accessible ($m \lesssim m_W$), so...

can one reconstruct the $\nu_R$ sector?

if SUSY is broken at $\Lambda > M_3$, then the high scale parameters ($Y_\nu$ and $M$) generate $[m_\nu]$, and contribute to $[m_\nu^2]$ via the RGEs.

There is a texture model independent parametrisation of the SUSY seesaw, that allows the renormalisable interactions of the $\nu_R$ to be reconstructed from weak-scale masses:

\[ [m_\nu], [m_\nu^2] \leftrightarrow Y_\nu, M \]

in practice: This reconstruction requires exact universality of the soft masses (!), and some contributions are unobservably small ($\ll$ exptal sensitivity)

in the non-SUSY case (Broncano et al) $Y_\nu$ and $M$ can be reconstructed from $[m_\nu]$ and dimension 6 operators $\propto M^{-2}$. 