Can one reconstruct the seesaw parameters?

(the masses, mixing matrix and Yukawa couplings of the ν_R)

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seesaw25 à Paris,
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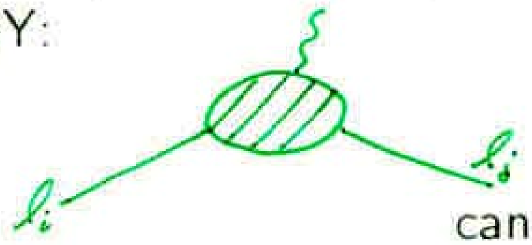
based mostly on JHEP 0109 (2001) 013, with A Ibarra

also: Nucl.Phys.B648 (2003) 345, JHEP 0303 (2003) 037, JHEP 0403 (2004) 020

Why is this an interesting question?

- the (3 ν_R, no triplet) seesaw today...
  - we observe small neutrino masses
  - "attractive" mechanism consistent with small m_ν

- the seesaw predicts
  - m_ν majorana $(0\nu 2\beta)$
  - if no SUSY (LHC?), then ~~flavour~~, ~~CP~~ $\propto m_\nu$:
    $\mu \to e\gamma$ small, EDMs small , ν mag. mos small
  - with SUSY:



can be observable

these are consistency checks

- to "test" a model, one needs
  1. determine model parameters from data
  2. make a prediction for an additional measurement
  3. confirm prediction

⇒ *can seesaw parameters be determined from data?*

NO

(realistic answer)

...but in the best of all (SUSY) dream worlds ??? *...yes...*

⇒ HOW? and WHY DOES IT WORK?

Outline

- parametrisations of the (type I) seesaw :
  - RH parameters (usual top-down)
  - LH parameters (? "bottom-up" ?)
  - (à la Casas-Ibarra)
  - fallacies thereof... the puppet show?

  $\Rightarrow$ what is special about the seesaw?

- "reconstruction" in SUSY
  - in principle (LH $\rightarrow$ RH)
  - in practise...

- "reconstruction" without SUSY? (Broncano et al)

- (leptogenesis as a "test"?)

- summary

input RH parameters

the usual seesaw parametrisation (3 ν_R, no triplet scalars): NB : no "texture" bases

1. 3 ν_R with masses M_i
2. a Yukawa $[Y_\nu]_{RL}$ with eigenvalues $\{y_1, y_2, y_3\}$
3. a unitary matrix $[V_R] : D_{Y_\nu} \leftrightarrow D_M$
4. Y_e eigenvalues $\{y_e, y_\mu, y_\tau\}$
5. a unitary matrix $V_L : D_{Y_e} \leftrightarrow D_{Y_\nu}$, (or, $U : D_{Y_e} \leftrightarrow D_{m_\nu}$) or maybe not

ν_R bases : D_{Y_ν}, D_M (V_R) $\quad$ ν_L bases : D_{Y_ν}, D_{Y_e} (V_L)

parameters: 9 real eigenvals, 6 angles, 6 phases

$$\Rightarrow [m_\nu] = V_L^T D_Y V_R^* D_M^{-1} V_R^\dagger D_Y V_L \langle H \rangle^2 \ (D_{Y_e}\ basis)$$
$$= U^* D_m U^\dagger$$

so in summary:

RH inputs: $D_M, [V_R], D_{Y_\nu}$, also LH inputs $[V_L], D_{Y_e}$

parameters: $12 + 9 = 21$.

can calculate: U, D_m

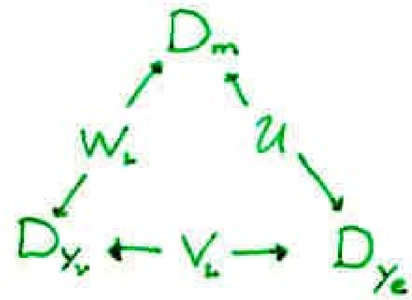
input LH parameters

the seesaw treats ν_L and ν_R symmetrically...so:

1. 3 ν_L with masses m_i
2. a Yukawa $[Y_\nu]_{RL}$ with eigenvalues $\{y_1, y_2, y_3\}$
3. a unitary matrix $[W_L] : D_{Y_\nu} \leftrightarrow D_m$
4. Y_e eigenvalues $\{y_e, y_\mu, y_\tau\}$
5. a unitary matrix $V_L : D_{Y_e} \to D_{Y_\nu}$, or, $U : D_{Y_e} \to D_{m\nu}$

ν_L bases : D_{Y_ν}, D_{Y_e}, D_m (ν_R basis : D_{Y_ν})
(parameters: 9 real eigenvals, 6 angles, 6 phases)

$$\Rightarrow M^{-1} = D_Y^{-1} W_L^* D_m W_L^\dagger D_Y^{-1}$$
$$= V_R^* D_M^{-1} V_R^\dagger$$



in summary:

LH inputs: D_m, $[W_l]$, D_{Y_ν}, U, D_{Y_e},
parameters: $12 + 9 = 21$
can calculate D_M, V_R

LH particle masses are $\lesssim m_W$ ...so...

1) do weak-scale observables depend on W_L $(= V_L U)$ and D_Y ?

2) is there any (realistic) hope of extracting V_L and D_Y from data?

conclusions one should not draw (d'après moi)

1. "leptogenesis is independent of LH parameters"

proof:

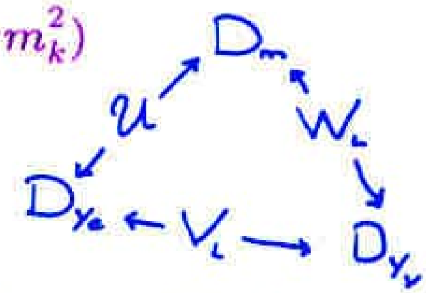
$$\epsilon = \sum_J \frac{\Im\{[V_R^\dagger D_Y^2 V_R]_{1J}^2\}}{[V_R^\dagger D_Y^2 V_R]_{11}} g\left(\frac{M_J^2}{M_1^2}\right)$$

no LH parameters appear. QED.

2. "light ν observables are independent of RH parameters"

proof:

$$P(\nu_\beta \to \nu_\alpha) = f(U_{\beta k}, U_{\alpha j}, ..., m_j^2 - m_k^2)$$



no RH parameters appear. QED.

1. D_m and U can be calculated from ν_R parameters, so obviously depend on them. Whereas leptogenesis does not care about Y_e, so does not care about U. Explicitly: U does not appear in the formula for ϵ in any LH parametrisation that includes $W_L : D_{Y_\nu} \leftrightarrow D_m$.

2. but — in the "top-down" parametrisation using (D_M, V_R, D_{Y_ν}, U, D_{Y_e}), V_R does not appear in $P(\nu_\beta \to \nu_\alpha)$.

1. specious argument! At $\Lambda \sim M$, D_{Y_e} and D_{Y_ν} are coupling constants in the LH sector. D_m is not. So a "correct" top-down parametrisation uses V_L not U. $\hookrightarrow \{D_m, Y_\nu = V_R^\dagger D_{Y_\nu} V_L, D_{Y_e}\}$

2. ah-ha! but at $\Lambda \sim m_W$ we know U and might get V_L in SUSY, so $W_L = V_L U$ is dependent in LH parametrisation. So $\epsilon(U)$ ...

? ?? ??? ???????

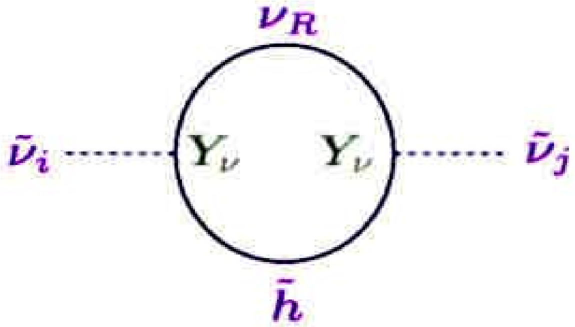
what is correct choice of "independent" parameters?

$$D_m \leftarrow \mathcal{U} \rightarrow D_{Y_e} \leftarrow V_L \rightarrow D_{Y_\nu}$$

Masiero + Borzumati

the SUSY See-Saw

- suppose soft scalar masses universal at M_{GUT}: $\sim m_o^2 \mathbf{I}$
- Renormalisation Group running will induce flavour violation at the weak scale in slepton masses:



$$\left[m_{\tilde{\nu}}^2\right]_{ij} \simeq (\text{diag part}) - \frac{3m_0^2 + A_0^2}{8\pi^2}(\mathbf{Y}_\nu^\dagger)_{ik}(\mathbf{Y}_\nu)_{kj} \log \frac{M_{GUT}}{M_k}$$

$Y_\nu^\dagger Y_\nu$ is the only source of lepton flavour violation in $[m_{\tilde{\nu}}^2]_{ij}$

$\Rightarrow$ extract $Y_\nu^\dagger Y_\nu \propto [V_L]^\dagger D_Y^2 [V_L]$ from $[m_{\tilde{\nu}}^2]$ *(in principle)*

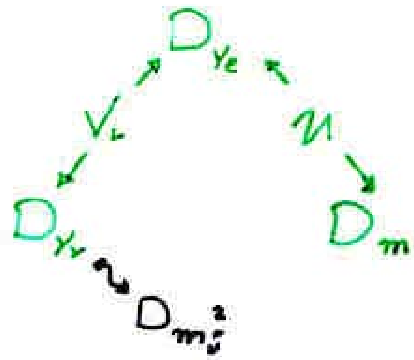
$\Rightarrow \{D_{Y_\nu}, V_L\}$ from sleptons, $\{D_{Y_e}, U, D_m\}$ from leptons

reconstruction in practise

apologies to people I should refer too!

known probably maybe ummm

from the leptons



- D_{Y_e} : m_τ, m_μ, m_e
  (could determine $\tan\beta$ in SUSY?)

- U : $\theta_{23}, \theta_{12}, \theta_{13}, \delta, \alpha, \beta$
  with a pure, high intensity ν beam: θ_{13}, δ?

- D_m : $m_3 > m_2 > m_1$
  know two Δm^2s, need pattern and absolute mass scale
  ? $0\nu 2\beta$, cosmology ?

from the sleptons

- V_L : $|[V_L]_{23}|, |[V_L]_{23}|, |[V_L]_{12}|$

  - *if* superpartners are "light" (observable at LHC, in $\ell_j \to \ell_i \gamma$)
    then from LFV, might extract $[\tilde m_{\tilde\nu}^2]_{\tau\mu}, [\tilde m_{\tilde\nu}^2]_{\mu e}, [\tilde m_{\tilde\nu}^2]_{\tau e}$

  - *if* ~~SUSY~~ at $\Lambda > M_3$, *then* $[\tilde m_{\tilde\nu}^2]_{\alpha\beta} \sim [V_L]_{\alpha 3}[V_L]^*_{\beta 3} y_3^2$.
    $\Rightarrow$ extract ?$|[V_L]_{23}|, |[V_L]_{13}|, y_3$
    ACK—other contributions to non-universal $[\tilde m_{\tilde\nu}^2]$???

- $D_{y\nu}$: y_3, y_2, y_1
  ...only y_3 makes significant contribution to RGEs

- 3 phases of V_L : $\varphi_1, \varphi_2, \varphi_3$
  CP in charged sleptons? ??EDMs ?? , ???$\tilde\nu$—$\bar{\tilde\nu}$ oscillations ???
  but there are other sources of CP that can contribute...

Summary

the (type I) seesaw treats ν_L and ν_R symmetrically, so can be described by masses, mixing angles and couplings of ν_R, or ν_L.

LH particles are kinematically accessible ($m \lesssim m_W$), so...
can one reconstruct the ν_R sector?

if SUSY is broken at at $\Lambda > M_3$, then the high scale parameters (Y_ν and M) generate $[m_\nu]$, and contribute to $[\tilde{m}^2_{\tilde{\nu}}]$ via the RGEs.
There is a *texture model independent* parametrisation of the SUSY seesaw, that allows the renormalisable interactions of the ν_R to be reconstructed from weak-scale masses:

$$[m_\nu], [m^2_{\tilde{\nu}}] \leftrightarrow Y_\nu, M$$

in practise: This reconstruction requires exact universality of the soft masses (!), and some contributions are unobservabley small ($\ll$ exptal sensitivity)

in the non-SUSY case (Broncano et al) Y_ν and M can be reconstructed from $[m_\nu]$ and dimension 6 operators $\propto M^{-2}$.