NEUTRINO MASSES IN EXTRA DIMENSIONS: A (VERY) BRIEF REVIEW

1. Low-scale gravity, brane-world models and (sub)mm dims.
   - Bulk neutrinos
   - multiple seesaw
2. Warped compactifications
3. Orbifold GUT's
4. Conclusions

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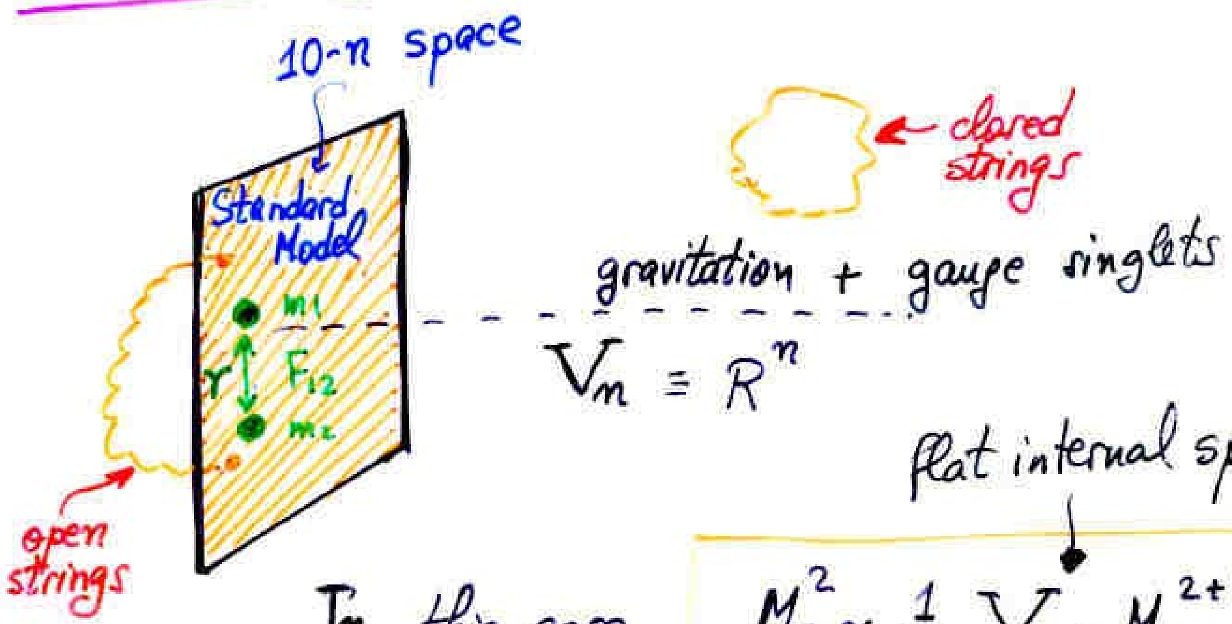
1) Low-scale strings: brane-world models, (sub)mm dimensions,

- String physics is accessible at future colliders only if $M_s \gtrsim \text{TeV} \ll M_P$.
  The appearance (Polchinski) of D-branes made this possible $\longrightarrow$ brane-world models

- ADD scenario

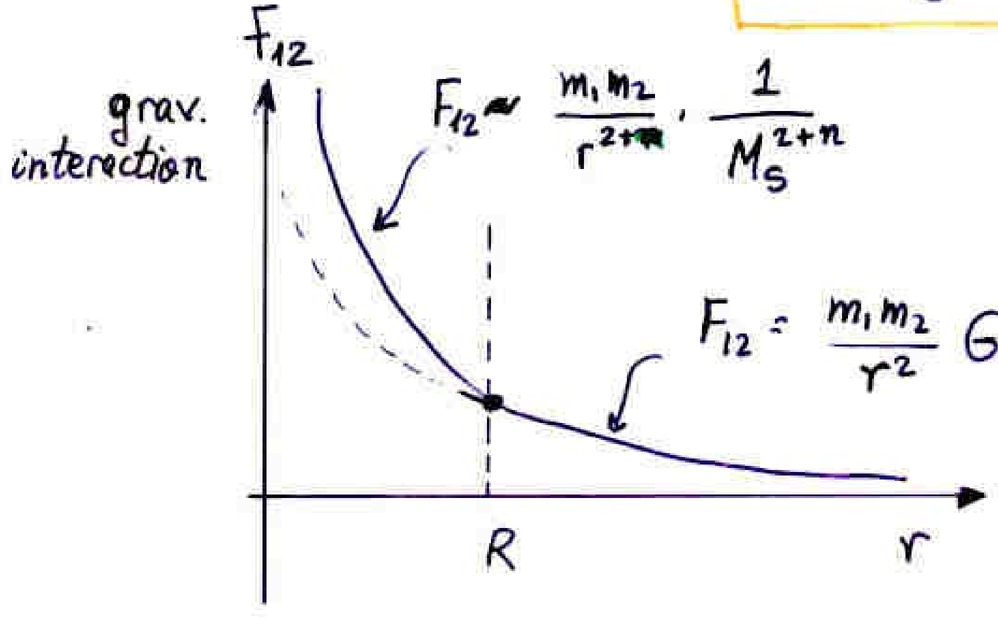


$$V_n \equiv R^n$$

flat internal space

In this case $$\boxed{M_P^2 \sim \frac{1}{g^4} V_n \cdot M_s^{2+n}}$$

$$\equiv V_n \cdot M_*^{2+n}$$



$\mathrm{TeV} \lesssim M_s \lesssim M_P$. If $M_s \sim \mathrm{TeV} \Rightarrow$

$n=1 \Rightarrow R \sim 10^{13}\,\mathrm{cm}$, excluded (solar system size)

$n=2 \Rightarrow R \sim \mathrm{mm}$

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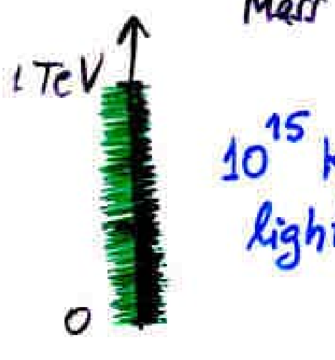
$n=6 \Rightarrow R \sim 10^{-13}\,\mathrm{cm}$

Present experimental limit (deviations from Newton universal attraction)

$$\boxed{R \lesssim 0.2\ \mathrm{mm}}$$

The scenario predicts that gravity becomes strong (string theory) at energy $M_s \sim \mathrm{TeV}$ and that there are very light graviton Kaluza-Klein states of mass $\gtrsim 10^{-3}\,\mathrm{eV}$.



10^{15} KK gravitons lighter than 1 TeV !

$\Rightarrow$ astrophysical and cosmological implications.

Potential problem of this scenario: unification of couplings, proton decay, neutrino masses

I Bulk right-handed neutrinos

(Diener, F.D. & Gherghetta; Arkani-Hamed, Dimopoulos, Dvali & March-Russell, 98; Dvali & Smirnov)

ν_L - sits on the Standard Model brane

ν_R - singlet (Dirac) fermion living in the "bulk"

H (Higgs) - on the Standard Model brane

- Dirac mass is very small (volume suppressed)

$$m_D \sim \frac{v}{(R M_*)^{\delta/2}} = v \left(\frac{M_*}{M_P}\right)^{\frac{\delta}{n}}$$

δ = nb. of dimensions felt by RH neutrino

$\delta \leq n$ $\qquad \boxed{\delta < n}$ needed if $M_* \sim \mathrm{TeV}$

What about Majorana masses and the Seesaw mechanism?

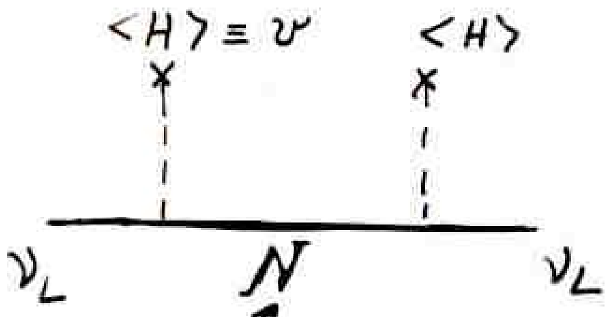
Two possibilities $\to$ brane Majorana masses i) ; bulk Majorana masses ii)

All cases, ν_R contains an infinity of Kaluza-Klein excitations which can participate directly and indirectly to oscillations and contribute to the ν_L neutrino masses.

(Mohapatra & coll., Ramond, Ross & coll., Valle & coll.)

TeV strings and neutrino physics

- conventional explanation of small neutrino masses in 4d:
  seesaw mechanism



$$\Rightarrow \quad \boxed{m_\nu \sim \frac{v^2}{M}}$$

Majorana neutrino, of mass M

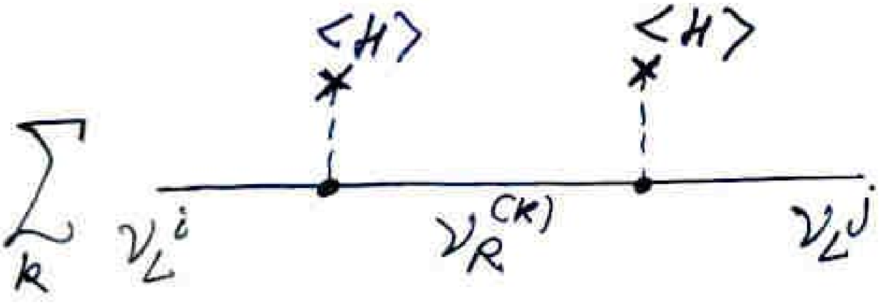
→ numerical values in the interesting range for

$$10^{12}\,\text{GeV} \lesssim M \lesssim 10^{15}\,\text{GeV},$$

natural in GUT and/or SUGRA theories.

Neutrino masses $\overset{?}{\longleftrightarrow}$ High-energy mass scales,

first evidence of physics beyond the Standard Model?



i) Brane Majorana masses M_0, $\delta = 1$

Physical masses λ are solutions of transcendental eq.

$$\boxed{\lambda \tan(\pi \lambda R) = \pi R \left(m_D^2 - \lambda M_0 \right)}$$

lowest eigenvalue λ_0 most important

$$M_0 \gg m_D \quad \Rightarrow \quad \lambda_0 \simeq \frac{m_D^2}{M_0}, \quad \text{standard seesaw}$$

- Both m_D and M_0 are volume suppressed but m_D^2/M_0 is ~~volume~~ independent

ii) Bulk Majorana masses M_0 (can be of 2 types: Lukas, Ramond, Romanino & Ross)

→ a higher-dimensional version of the seesaw mechanism is at work for $M_0 \gg R^{-1}$

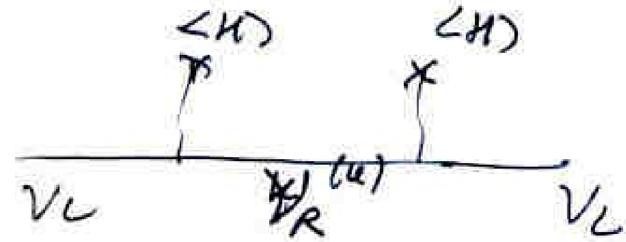
$$m_\nu^{\text{naive}} \sim \frac{m_D^2}{M_0} \longrightarrow m_D^2 \cdot R \, ,$$

replacement $\quad M_0 \to \frac{1}{R}$

Ex: "vector" Majorana mass $M_0 \overline{\Psi^c} \gamma_5 \Psi$, masses determined by

$$\boxed{\lambda \tan\left[\pi R (M_0 - \lambda)\right] = -\pi R m_D^2}$$

M_0 only matters modulo $\frac{\text{integer}}{R}$



$\Rightarrow$

$$m_\nu \sim m_D^2 \sum_n \left(\frac{1}{\frac{k}{R} - M_0} - \frac{1}{\frac{n}{R} + M_0} \right) \sim m_D^2 R \, ,$$

since always a light state $\frac{k}{R} - M_0 \sim \mathcal{O}\left(\frac{1}{R}\right)$

For more than one relevant dimension, the active / sterile mixing is not dominated by the lightest KK modes and the mixing with the heaviest modes does not lead to oscillations.

In this case, after integrating out heavy KK modes
=> dimension-six operators

$$\mathcal{L}_{tree} \sim \varepsilon_{ij} \, G_F \, (H^+ \bar{L}_i) \, i\partial\!\!\!/ \, (H L_j)$$

generating flavour transitions (Giudice et al, 01)

II Multiple seesaw mechanisms for $M_* \gtrsim 100\,\mathrm{TeV}$ but no bulk fields

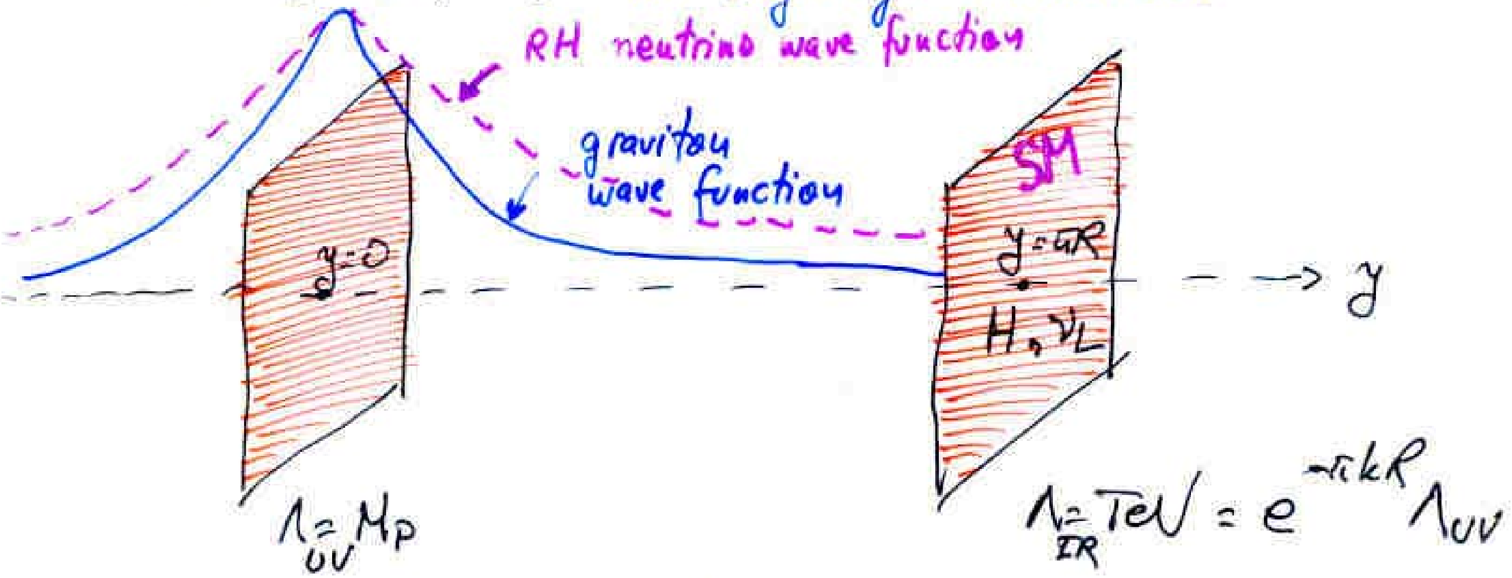
$$m_\nu \sim \frac{v^4}{M_*^3} ,$$

need additional fields, but complete model possible to construct (ED & C. Savoy, 02)

fermion masses $\longleftrightarrow$ $\sin^2\theta_W\,(M_*) = \frac{1}{4}$

2. Warped compactifications (Randall & Sundrum)

- the metric has a strong dependence in the internal space, producing hierarchies and (graviton) localization, no very large dimensions



$$ds_5^2 = dy^2 + \underbrace{e^{-2k|y|}}_{\text{warp factor}} dx_4^2$$

$$M_P^2 \sim \frac{1}{k} M_*^3$$

- If SM fields (at least the Higgs) localized on TeV brane: brane fields or bulk fields trapped on the TeV brane, hierarchy "solved".

- Right-handed neutrino in the bulk (Grossman-Neubert, 99)

$$\Psi = \begin{pmatrix} \nu_R \\ \bar{\mu}_R \end{pmatrix}$$

Localization of the RH neutrino on the Planck brane is done by adding a Dirac bulk mass

$$- m \, \varepsilon(y) \, \bar{\Psi} \Psi$$

giving rise to a RH neutrino wave function

$$\nu_R(y) \sim e^{(k-2m) \cdot |y|}$$

$m > \frac{k}{2} \Rightarrow$ localization on the Planck brane, very small Dirac neutrino masses

- tower of TeV scale Kaluza-Klein masses for the bulk neutrino

3. Orbifold GUT's

→ Higher-dimensional Grand Unified Theories,
gauge group $G = SU(5), SO(10), \dots$

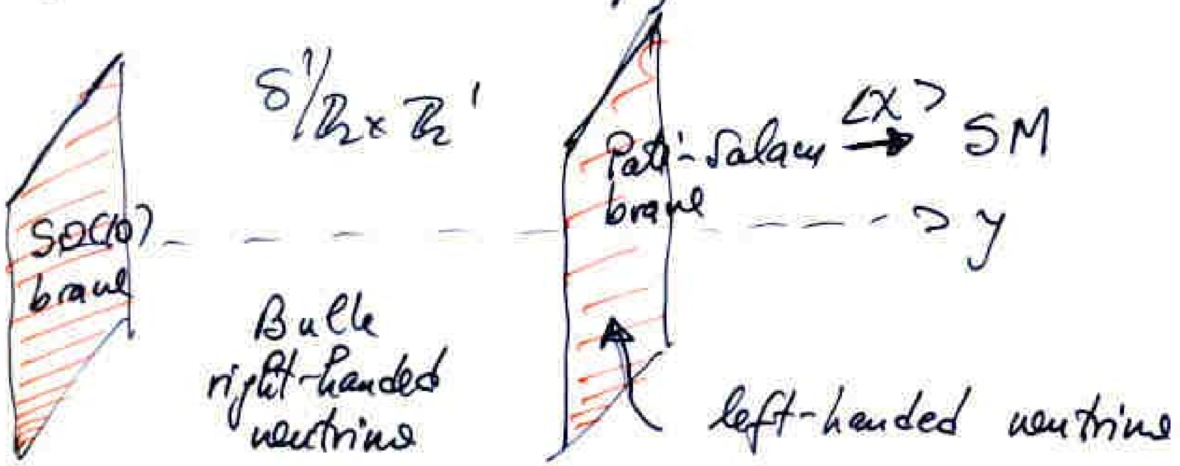
$G \xrightarrow[\text{(boundary conditions)}]{\text{compactification}}$ Standard Model, Pati-Salam group

Most studied example $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$,
where $\mathbb{Z}_2$ breaks $\mathcal{N}=2$ to $\mathcal{N}=1$ SUSY
$\mathbb{Z}_2'$ breaks $SU(5) \to SU(3)\times SU(2) \times U(1)$
$SO(10) \to SU(4)_c \times SU(2)_L \times SU(2)_R$

By construction, only MSSM (Pati-Salam) fields have $(+,+)$ parities and have zero-modes. All the other fields are heavy $M \sim R^{-1} \sim 10^{14}\,\text{GeV}$,
gauge coupling unification basically as in SUSY SU(5)
→ SM, unification scale $\sim 10^{17}$ GeV.
(Kawamura, Hall & Nomura, ... 00-01)

Neutrino masses here need a seesaw mechanism
(Raby & coll.)

Ex (from Kim & Raby)



Bulk Majorana mass $M_0 \gg R^{-1} \Rightarrow$ effective seesaw scale $M_{eff} \sim R^{-1} \Rightarrow$

$$m_\nu \sim \frac{m_D^2}{M_{eff}}$$, works well for

ν_τ with $R^{-1} \sim 10^{14}$ GeV

CONCLUSIONS

- Seesaw mechanism still very important in orbifold GUT's
- A higher-dimensional seesaw mechanism at work $M_{eff} \sim \frac{1}{R}$
- New ways of getting naturally small Dirac masses